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Status of neutrino masses and mixing and future perspectives

S M Bilenky

Joint Institute for Nuclear Research, Dubna, R-141980, Russia
and
Scuola Internazionale Superiore di Studi Avanzati, I-34014 Trieste, Italy

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Abstract

The status of the problem of neutrino masses, mixing and oscillations is discussed. Future perspectives are briefly considered.

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1. Introduction

Evidence for neutrino oscillations obtained in the Super-Kamiokande [1], SNO [2], KamLAND [3] and other neutrino experiments [4] is one of the most important recent discoveries in particle physics. There is no natural explanation of the smallness of neutrino masses in the standard model. A new, beyond the standard model mechanism of the generation of neutrino masses is necessary in order to explain experimental data. In this paper we will discuss (i) basics of neutrino mixing and neutrino oscillations, (ii) evidence for neutrino oscillations and (iii) open problems and future perspective.

2. Basics of neutrino mixing and oscillations

There are three basic ingredients in the theory of neutrino oscillations: neutrino interaction, neutrino mass term and neutrino transition probabilities.

Neutrino interaction is well known. All existing experimental data are perfectly described by Lagrangians of the charged current and neutral current interactions of the standard model:

$$\mathcal{L}_I^{\text{CC}} = -\frac{g}{2\sqrt{2}}j_\alpha^{\text{CC}}W^\alpha + \text{h.c.}, \quad \mathcal{L}_I^{\text{NC}} = -\frac{g}{2\cos\theta_W}j_\alpha^{\text{NC}}Z^\alpha. \quad (1)$$

Here,

$$j_\alpha^{\text{CC}} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_\alpha l_L, \quad j_\alpha^{\text{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_\alpha \nu_{lL} \quad (2)$$

are leptonic charged current and neutrino neutral current, g is $SU(2)$ gauge constant and θ_W is the weak angle.

Neutrino mass term determines neutrino masses, neutrino mixing and nature of massive neutrinos. For neutrinos, particles with electric charge equal to zero, there are two general possibilities for the mass terms (see, for example, [5]).

2.1. Majorana mass term

Let us introduce the column of the left-handed fields

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \end{pmatrix}. \quad (3)$$

The fields ν_{sL} do not enter into the Lagrangians of the standard CC and NC interaction (1). For this reason, they are called sterile fields. The conjugated column

$$(n_L)^c = C(\bar{n}_L)^T (C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, C^T = -C) \quad (4)$$

is a right-handed column. The most general Majorana mass term has the form

$$\mathcal{L}^M = -\frac{1}{2}\bar{n}_L M^M (n_L)^c + \text{h.c.} = -\frac{1}{2}\bar{n}_L M^M C(\bar{n}_L)^T + \text{h.c.}, \quad (5)$$

where M^M is a symmetrical, complex matrix. We have

$$M^M = U m U^T, \quad (6)$$

where U is a unitary matrix and $m_{ik} = m_k \delta_{ik}$, $m_k > 0$. From (5) and (6) for the mass term, we find

$$\mathcal{L}^M = -\frac{1}{2}\bar{\nu} m \nu = -\frac{1}{2} \sum_{k=1}^{3+n_s} m_k \bar{\nu}_k \nu_k, \quad (7)$$

where

$$\nu = U^\dagger n_L + (U^\dagger n_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \end{pmatrix}. \quad (8)$$

From (8), we have

$$\nu_k^c = \nu_k \quad (k = 1, 2, 3, \dots, 3 + n_s), \quad (9)$$

where n_s is the number of sterile fields. Thus, ν_k is the field of Majorana neutrinos with mass m_k . From (3) and (8) for the mixing we have

$$\nu_{lL} = \sum_{k=1}^{3+n_s} U_{lk} \nu_{kL}, \quad \nu_{sL} = \sum_{k=1}^{3+n_s} U_{sk} \nu_{kL}. \quad (10)$$

2.2. Dirac mass term

We will now introduce columns of *independent* left-handed and right-handed fields:

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s1L} \\ \vdots \end{pmatrix}, \quad n_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \\ \nu_{s1R} \\ \vdots \end{pmatrix}. \quad (11)$$

The most general Dirac neutrino mass term has the form

$$\mathcal{L}^D = -\bar{n}_L M^D n_R + \text{h.c.} \quad (12)$$

The complex matrix M^D can be presented in the form

$$M^D = U m V^\dagger, \quad (13)$$

where V and U are unitary matrices and $m_{ik} = m_k \delta_{ik}$, $m_k > 0$. From (12) and (13) for the Dirac mass term, we find

$$\mathcal{L}^D = -\bar{\nu} m \nu = -\sum_{k=1}^{3+n_s} m_k \bar{\nu}_k \nu_k, \quad (14)$$

where

$$\nu = U^\dagger n_L + V^\dagger n_R = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \end{pmatrix}. \quad (15)$$

From (14) and (15), it follows that $\nu_k(x)$ is the Dirac field of neutrinos ($L = 1$) and antineutrinos ($L = -1$) (L is the conserved total lepton number). From (15), for the neutrino mixing, we will obtain equation (10) in which ν_k is the field of the Dirac neutrino with mass m_k .

The type of the neutrino mass term at present is unknown. We will mention here only that in the case of the most popular see-saw mechanism of neutrino mass generation, the neutrino mass term is given by (5) with $\nu_{s1L} = (\nu_{eR})^c$, $\nu_{s2L} = (\nu_{\mu R})^c$ and $\nu_{s3L} = (\nu_{\tau R})^c$ where 1, 2, 3 are indices of s . It is called Dirac and Majorana mass terms and is a sum of left-handed and right-handed Majorana and Dirac mass terms for three left-handed and right-handed neutrino fields. If the left-handed Majorana matrix is equal to zero and the right-handed Majorana matrix is much larger than the Dirac mass matrix, in this case, massive neutrinos are Majorana particles with masses which are much smaller than the masses of leptons or quarks.

We will now consider transitions of flavour neutrinos in vacuum. The state of flavour neutrino ν_l with momentum \vec{p} produced in a CC weak process together with l^+ is given by coherent superposition of states of neutrinos with different masses $|\nu_i\rangle$ (see [5]):

$$|\nu_l\rangle = \sum_k U_{lk}^* |\nu_k\rangle. \quad (16)$$

After time t , for the neutrino state, we have

$$|\nu_l\rangle_t = \sum_k e^{-iE_k t} U_{lk}^* |\nu_k\rangle = e^{-iE_l t} \sum_{k=1}^{3+n_s} e^{-i\frac{\Delta m_{lk}^2}{2E} t} U_{lk}^* |\nu_k\rangle, \quad (17)$$

where $\Delta m_{ik}^2 = m_k^2 - m_i^2$. Neutrinos are detected via observation of CC and NC processes in which flavour neutrinos ν_l take part. Existing neutrino oscillation data are described by the minimal scheme of the three-neutrino mixing. In the case $n_s = 0$, from (17), we find

$$|\nu_l\rangle_t = e^{-iE_l t} \sum_{l'} |\nu_{l'}\rangle \sum_{k=1}^3 U_{l'k} e^{-i \frac{\Delta m_{ik}^2}{2E} t} U_{lk}^*. \quad (18)$$

From this expression, for the probability of the transition $\nu_l \rightarrow \nu_{l'}$, we find the following standard expression:

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \delta_{ll'} + \sum_{k=2,3} U_{l'k} (e^{-i \Delta m_{ik}^2 \frac{L}{2E}} - 1) U_{lk}^* \right|^2, \quad (19)$$

where $L \simeq t$ is the distance between neutrino production and detection points. Transition probabilities depend on six parameters (two neutrino mass-squared differences Δm_{12}^2 and Δm_{23}^2 , three mixing angles θ_{12} , θ_{23} and θ_{13} and one CP phase δ) and have a rather complicated form. However, two parameters are small:

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq \frac{1}{30}, \quad \sin^2 \theta_{13} \leq 5 \times 10^{-2}. \quad (20)$$

In the leading approximation in which the contribution of small parameters is neglected, neutrino oscillations are described by simple two-neutrino expressions [5]. In fact, let us consider atmospheric and long baseline accelerator experiments with $\Delta m_{23}^2 \frac{L}{2E} \gtrsim 1$. Neglecting the contribution of small quantities $\Delta m_{12}^2 \frac{L}{2E}$ and $\sin^2 \theta_{13}$, we can see that dominant oscillations in the atmospheric region of $\frac{L}{E}$ are $\nu_\mu \leftrightarrow \nu_\tau$. The probability of ν_μ to survive is given by the standard two-neutrino expression

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - P(\nu_\mu \rightarrow \nu_\tau) = 1 - \frac{1}{2} \sin^2 2\theta_{23} \left(1 - \cos \Delta m_{23}^2 \frac{L}{2E} \right), \quad (21)$$

which depend only on two oscillation parameters $\sin^2 2\theta_{23}$ and Δm_{23}^2 , respectively.

Let us now consider the reactor KamLAND experiment and solar neutrino experiments. For these experiments, small Δm_{12}^2 is relevant and the contribution to the transition probabilities of 'large' Δm_{23}^2 is averaged. Neglecting $\sin^2 \theta_{13}$ for $\bar{\nu}_e \rightarrow \bar{\nu}_e$ survival probability in vacuum (the KamLAND experiment), we find the following expression:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{12} \left(1 - \cos \Delta m_{12}^2 \frac{L}{2E} \right). \quad (22)$$

For solar neutrinos MSW matter effect must be taken into account. Neglecting the contribution of $\sin^2 \theta_{13}$, we will find that $\nu_e \rightarrow \nu_e$ transition probability is given by two-neutrino expression

$$P(\nu_e \rightarrow \nu_e) = P_{\text{mat}}^{(1,2)}(\sin^2 \theta_{12}, \Delta m_{12}^2, \rho_e), \quad (23)$$

where ρ_e is electron number density. Thus, in the leading approximation *decoupling of oscillations in atmospheric-LBL and solar-KamLAND regions* takes place. Existing experimental data are in agreement with such a picture of neutrino oscillations.

3. Evidence for neutrino oscillations

First model-independent evidence for neutrino oscillations was obtained in the Super-Kamiokande atmospheric neutrino experiment [1]. In this experiment, atmospheric ν_e and ν_μ

are detected in the large 50 kt water Cherenkov detector. If there are no neutrino oscillations, the number of electron and muon events must satisfy the following relation:

$$N_l(\cos\theta) = N_l(-\cos\theta) \quad (l = e, \mu). \quad (24)$$

Here, θ is the zenith angle. Significant violation of this symmetry relation was observed in the case of high-energy muon events. For the ratio of the total number U of up-going muons ($500 \lesssim L \lesssim 13000$ km) and total number D of the down-going muons ($20 \lesssim L \lesssim 500$ km), it was found

$$\left(\frac{U}{D}\right)_\mu = 0.551 \pm 0.035 \pm 0.004. \quad (25)$$

In the Super-Kamiokande experiment, the $\frac{L}{E}$ dependence of the ν_μ survival probability was measured. From (21) it follows that the survival probability has a first minimum at $\Delta m_{23}^2 \frac{L}{2E} = \pi$. This minimum was clearly demonstrated by the data. From the analysis of the Super-Kamiokande data, the following 90% CL ranges of the oscillation parameters were found:

$$1.9 \times 10^{-3} \leq \Delta m_{23}^2 \leq 3.1 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} > 0.9. \quad (26)$$

For the best-fit values of the parameters, it was obtained

$$\Delta m_{23}^2 = 2.5 \times 10^{-3} \text{ eV}^2; \quad \sin^2 2\theta_{23} = 1. \quad (\chi^2/\text{dof} = 839.7/755). \quad (27)$$

The evidence for ν_μ disappearance, obtained in the Super-Kamiokande atmospheric neutrino experiment, was confirmed by accelerator K2K [6] and MINOS [7] experiments. In the K2K experiment, ν_μ 's produced at the KEK accelerator are detected by the Super-Kamiokande detector at a distance of about 250 km. 112 ν_μ events were observed in the experiment. In the case of no neutrino oscillations, $158.1_{-8.6}^{+9.2}$ events were expected. From the analysis of the K2K results for the best-fit values of the oscillation parameters, it was found the values

$$\Delta m_{23}^2 = 2.64 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} = 1, \quad (28)$$

which are compatible with the Super-Kamiokande 90% CL ranges (26).

In the MINOS long-baseline accelerator experiment (Fermilab–Soudan, 730 km), the number of expected $\nu_\mu + \bar{\nu}_\mu$ events is 298 ± 15 . The number of observed events is equal to 204. From the analysis of the data, the following best-fit values of the oscillation parameters were obtained:

$$\Delta m_{23}^2 = (3.05_{-0.55}^{+0.60} \pm 0.12) 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} = 0.88_{-0.15}^{+0.12} \pm 0.06. \quad (29)$$

In all solar neutrino experiments (Homestake, GALLEX-GNO, SAGE and Super-Kamiokande [4]), observed rates are 2–3 times smaller than the rates predicted by SSM [8]. Model-independent evidence for neutrino oscillations was obtained in the solar SNO experiment [2]. In this experiment, solar neutrinos are detected via the observation of three reactions

$$\nu_e + d \rightarrow e^- + p + p \text{ (CC)}, \quad \nu_x + d \rightarrow \nu_x + n + p \text{ (NC)}, \quad \nu_x + e \rightarrow \nu_x + e \text{ (ES)}. \quad (30)$$

From the measurement of the CC rate, the flux of the solar ν_e on the earth can be inferred. From the measurement of the NC rate, the flux of all active neutrinos ν_e , ν_μ and ν_τ can be determined. In the SNO experiment, it was obtained

$$\begin{aligned} \Phi_{\nu_e}^{\text{SNO}} &= (1.68 \pm 0.06 \pm 0.09) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \\ \Phi_{\nu_{e,\mu,\tau}}^{\text{SNO}} &= (4.94 \pm 0.21 \pm 0.38) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}. \end{aligned} \quad (31)$$

The flux of ν_e on the earth is about three times smaller than the total flux of ν_e , ν_μ and ν_τ :

$$\frac{\Phi_{\nu_e}^{\text{SNO}}}{\Phi_{\nu_e, \mu, \tau}^{\text{SNO}}} = 0.340 \pm 0.023 \pm 0.031. \quad (32)$$

Thus, it was proved by the SNO experiment that the solar ν_e 's on the way from the sun to the earth are transformed into ν_μ and ν_τ .

The total flux of ν_e , ν_μ and ν_τ , measured by the SNO experiment, is in agreement with the flux predicted by SSM [8]:

$$\Phi_{\nu_e}^{\text{SSM}} = (5.69 \pm 0.91) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}. \quad (33)$$

Model-independent evidence for neutrino oscillations was found in the reactor KamLAND experiment. In this experiment, $\bar{\nu}_e$'s from 53 reactors in Japan are detected via the observation of the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ by a detector in the Kamiokande mine. The average distance from reactors to the detector is about 170 km. The experiment is sensitive to Δm_{12}^2 and $\sin^2 \theta_{12}$. The expected (without oscillations) number of events in the KamLAND experiment is 365.2 ± 23.7 . The observed number of events is 258. The ratio of the observed and expected events is equal to $R = 0.658 \pm 0.044 \pm 0.047$. Significant distortion of the spectrum of positrons produced in the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ was observed in the KamLAND experiment. From the global analysis of solar and KamLAND data for neutrino oscillation parameters, it was found the values

$$\Delta m_{12}^2 = 8.0_{-0.4}^{+0.6} 10^{-5} \text{ eV}^2, \quad \tan^2 \theta_{12} = 0.45_{-0.07}^{+0.09}. \quad (34)$$

Summarizing, model-independent evidence for neutrino oscillations driven by small neutrino masses and neutrino mixing were obtained in neutrino experiments. All data (except LSND) are in agreement with the three-neutrino mixing. Four neutrino oscillation parameters are determined from the analysis of oscillation data with approximate accuracies:

$$\Delta m_{12}^2 (\sim 10\%), \quad \tan^2 \theta_{12} (\sim 20\%), \quad \Delta m_{23}^2 (\sim 25\%) \quad \text{and} \quad \sin^2 2\theta_{23} (\sim 30\%). \quad (35)$$

For the parameter $\sin^2 \theta_{13}$, only CHOOZ bound ($\sin^2 \theta_{13} \lesssim 5 \times 10^{-2}$) exists. The CP phase δ is unknown.

4. Future perspectives

For further progress, the following basic questions must be answered.

4.1. Are neutrinos with definite mass Majorana or Dirac particles?

The most sensitive way to study the nature of massive neutrinos is the search for neutrinoless double β -decay

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \quad (36)$$

of ^{76}Ge , ^{130}Te , ^{136}Xe , ^{100}Mo and other even-even nuclei. The matrix element of $0\nu\beta\beta$ -decay is proportional to the effective Majorana mass $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$.

From the best existing lower bounds on the half-lives of the $0\nu\beta\beta$ -decay

$$\begin{aligned} T_{1/2}^{0\nu}({}^{76}\text{Ge}) &\geq 1.9 \times 10^{25} \text{ years (Heidelberg-Moscow)} \\ T_{1/2}^{0\nu}({}^{130}\text{Te}) &\geq 5.5 \times 10^{23} \text{ years (Cuoricino)} \end{aligned} \quad (37)$$

for the effective Majorana mass the following bounds $|m_{\beta\beta}| \leq (0.2-1.2) \text{ eV}$ can be inferred. Many new experiments on the search for $0\nu\beta\beta$ -decay (CUORE, GERDA, EXO, MAJORANA and others) are now in preparation [9]. The goal of the future experiments is to reach the sensitivity $|m_{\beta\beta}| \simeq \text{afew } 10^{-2} \text{ eV}$.

4.2. What is the value of the lightest neutrino mass m_0 ?

From the measurement of the β -spectrum of ${}^3\text{H}$ in Mainz and Troitsk experiments [10, 11] it was obtained $m_0 < 2.3$ eV. In the future KATRIN experiment [12], the sensitivity $m_0 \simeq 0.2$ eV is planned to be reached.

From cosmological data for the sum of the neutrino masses, it was found the range $\sum_i m_i < (0.2\text{--}0.7)$ eV. The future data will be sensitive to $\sum_i m_i \simeq 5 \times 10^{-2}$ eV (see [13]).

4.3. What is the value of the parameter $\sin^2 \theta_{13}$?

If the value of this parameter is not too small, it will be possible to study such fundamental effects of the three-neutrino mixing as CP violation in the lepton sector and to reveal the character of neutrino mass spectrum. The existing neutrino oscillation data are compatible with two types of neutrino mass spectra: $m_1 < m_2 < m_3$, $\Delta m_{12}^2 \ll \Delta m_{23}^2$ (normal spectrum) and $m_3 < m_1 < m_2$, $\Delta m_{12}^2 \ll |\Delta m_{13}^2|$ (inverted spectrum), respectively. New reactor experiments DOUBLE CHOOZ and Daya Bay [14] are in preparation at present. In these experiments, correspondingly, factor 10 and 20 improvement in the sensitivity to $\sin^2 \theta_{13}$ is expected. In the accelerator T2K [15] experiment, sensitivity to $\sin^2 \theta_{13}$ will be about 25 times better than in the CHOOZ experiment. Future neutrino facilities (Super beam, β -beam, Neutrino factory [16]) will allow us to study the phenomenon of neutrino oscillations with very high precision. There is no doubt that the accomplishment of this experimental programme will be crucial for understanding the origin of neutrino masses and mixing.

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